How appropriate is optimism for Interface Automata

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D-Con 2013

Interface-based design

- Interfaces abstract from details like data
- Focus on behavioural type and communication
- Analysis of errors in communication
- Refinement-/Implementation-relation
  - Refinement of components → refinement of system
  - → precongruence

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- Automata with disjoint inputs, outputs and internal actions
- Composition of 'compatible' Automata ($I_1 \cap I_2 = \emptyset = O_1 \cap O_2$)
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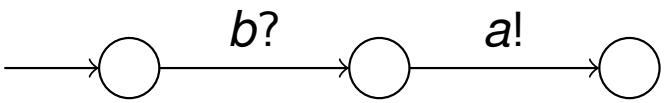

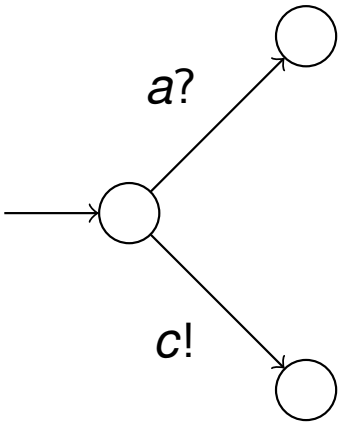

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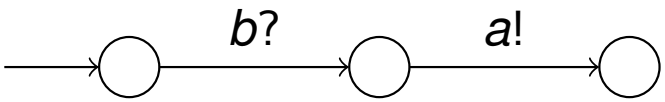

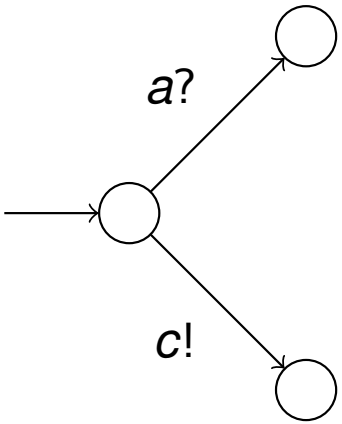

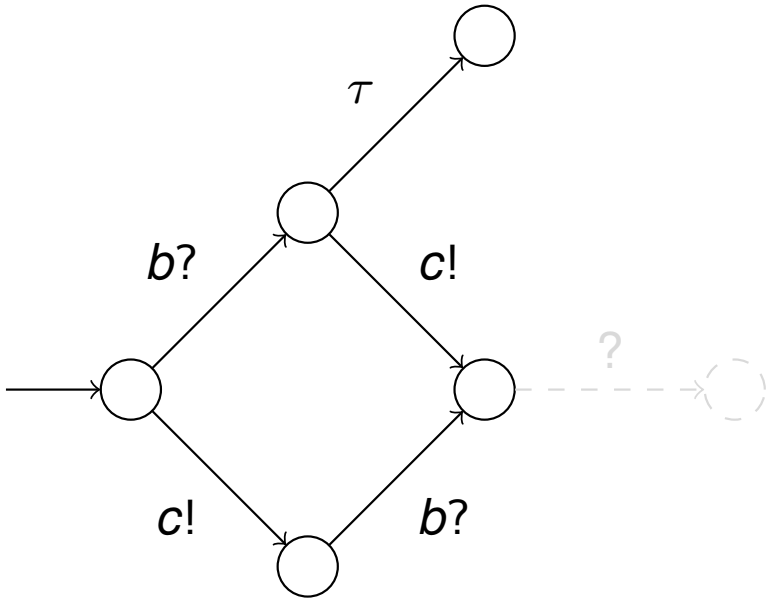

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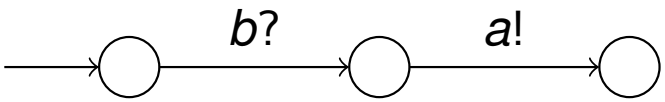
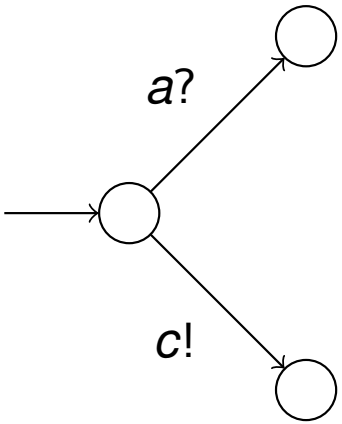
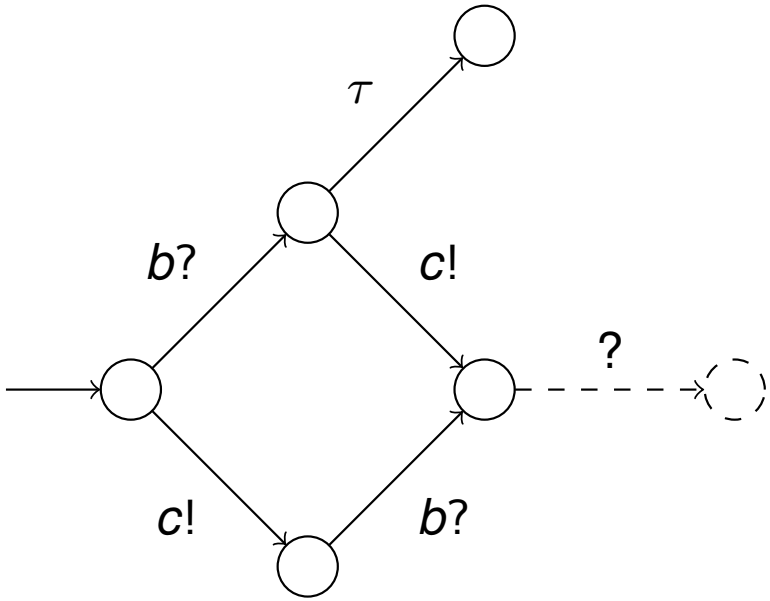
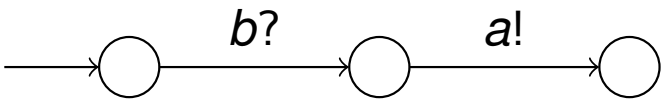
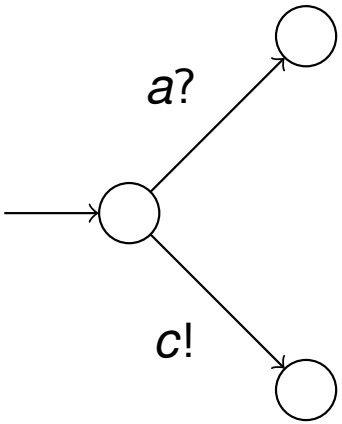


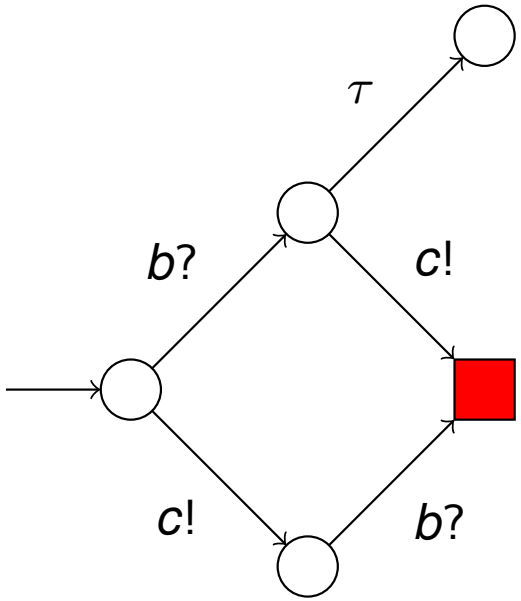




Composition – Errors







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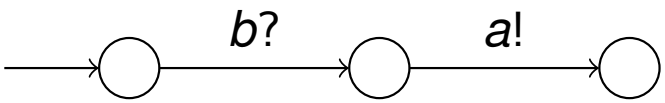
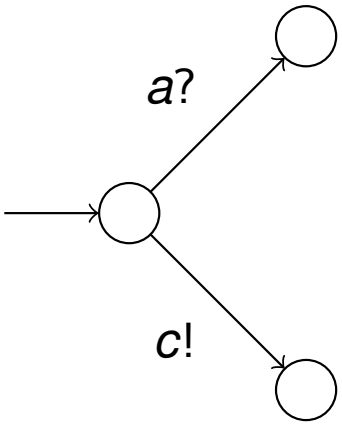
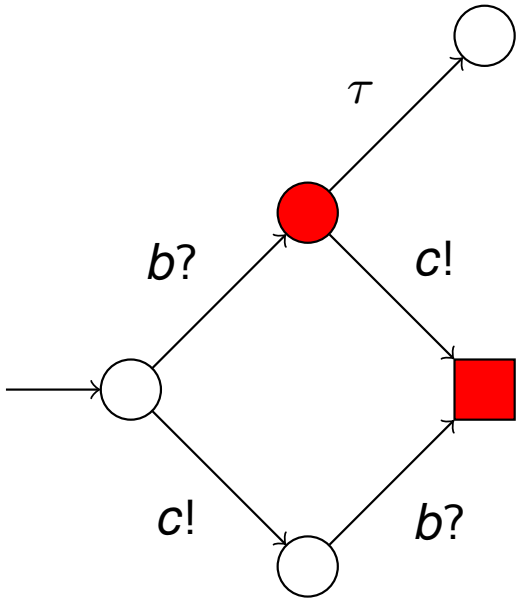
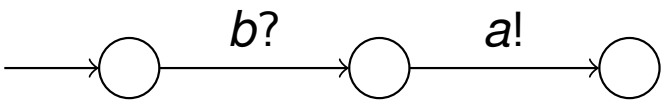
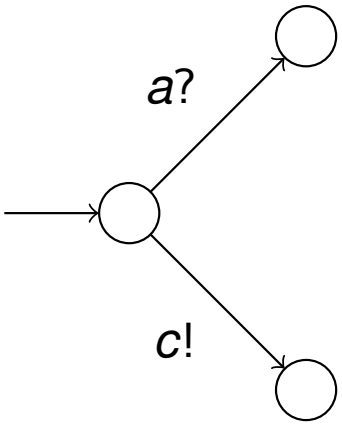


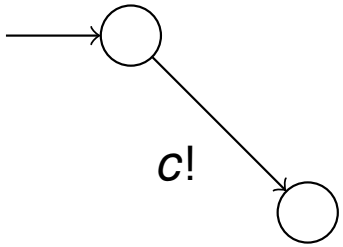




Composition – Errors / Pruning







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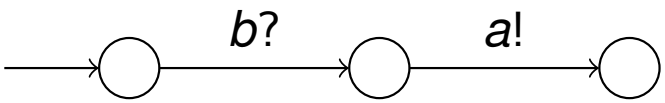
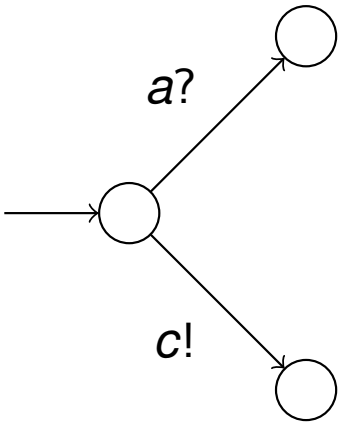
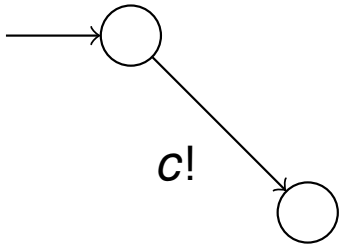
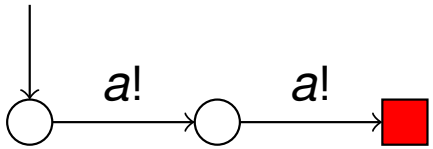






works for input determinate systems

Unavoidable Error Lemma



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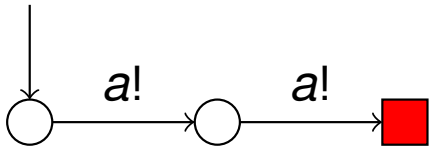

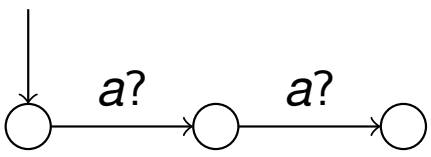

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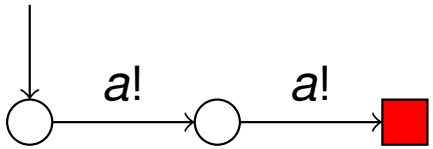

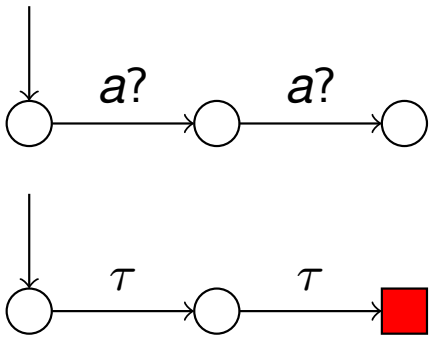

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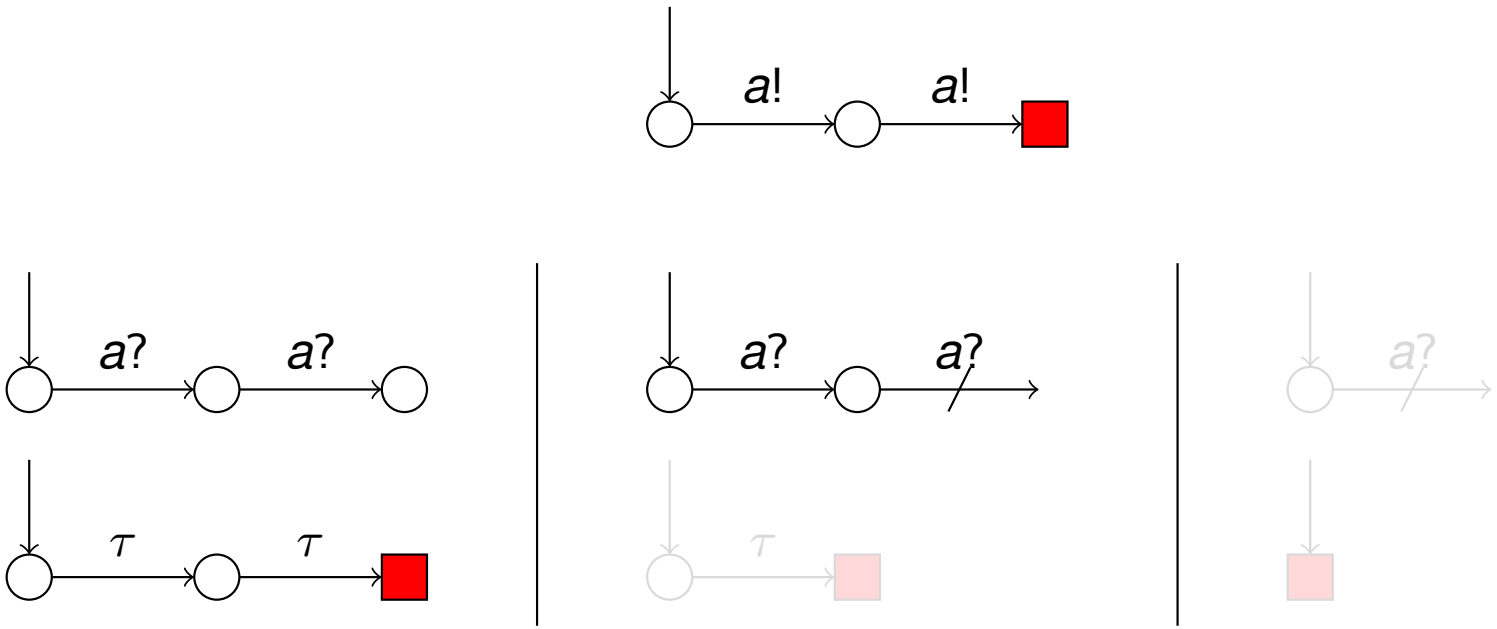

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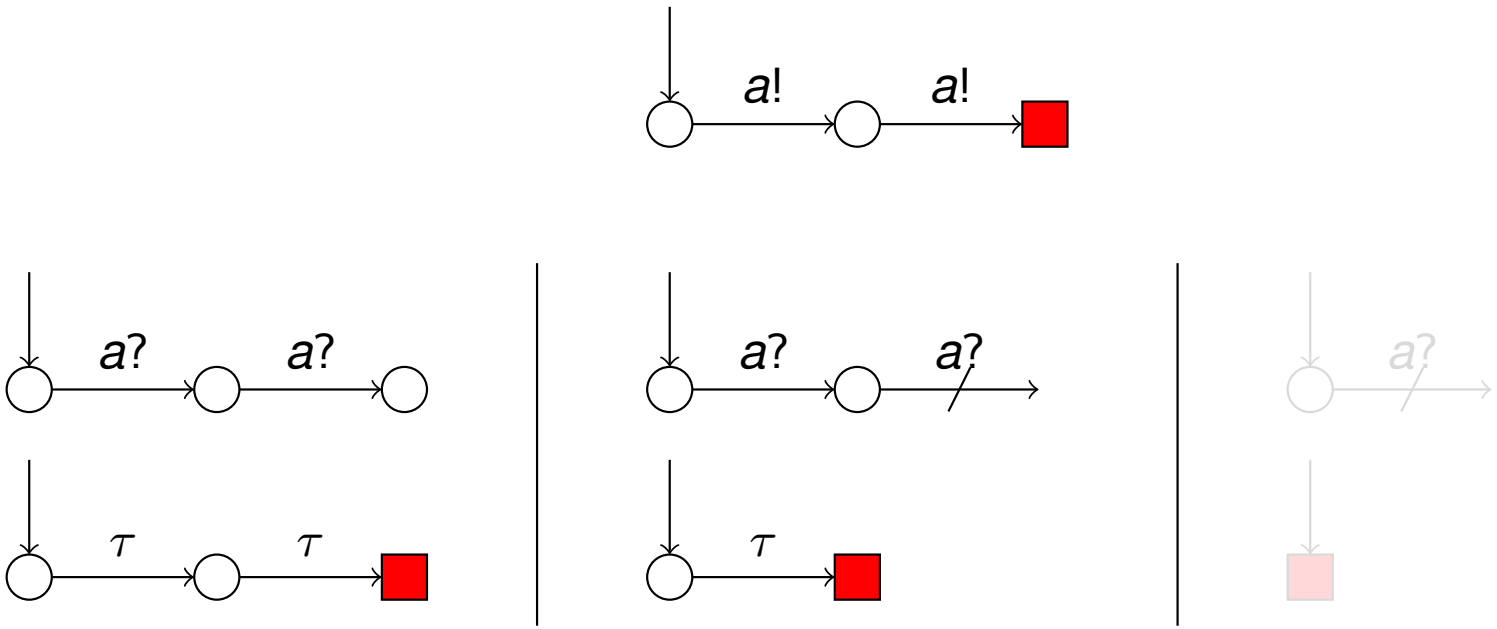

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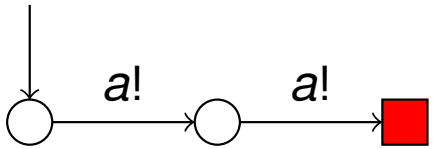

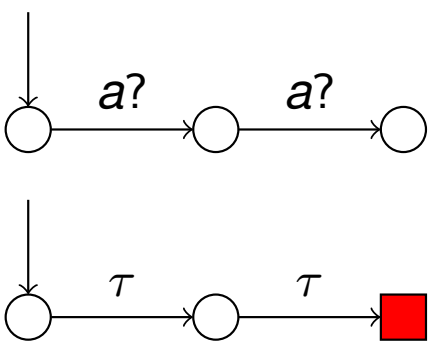

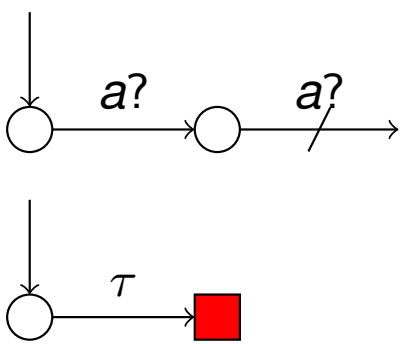

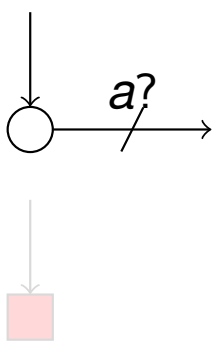

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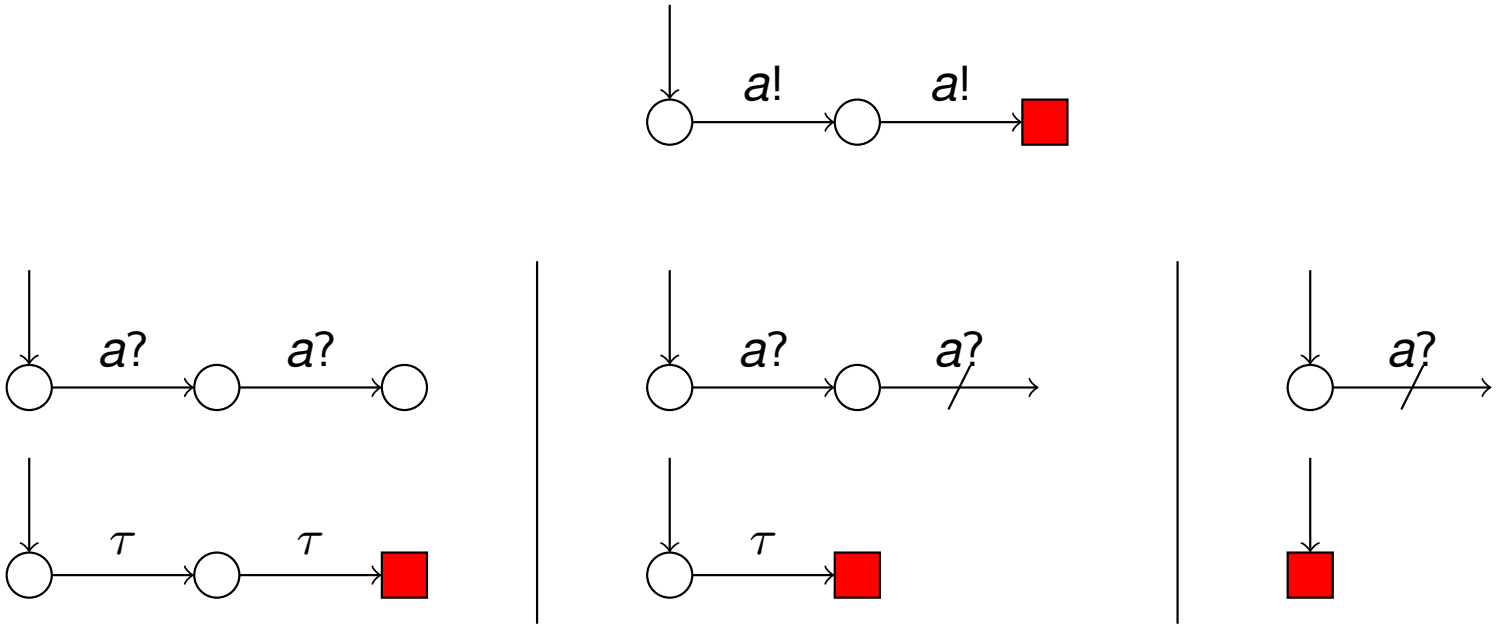

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Precongruence

A preorder relation $\sqsubseteq$ is a precongruence w.r.t. $\mid$ if

$$\mathit{Impl} \sqsubseteq \mathit{Spec} \quad \Rightarrow \quad \mathit{Impl} \mid \mathit{Env} \sqsubseteq \mathit{Spec} \mid \mathit{Env}$$

Refinement / Implementation

Basic requirement:
If the specification is error free, then the implementation has to be error free as well

Coarsest precongruence: $\sqsubseteq^{C}$

- Fulfilling the basic requirement
- Precongruence w.r.t. $\|$
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Functions

- $prune: \Sigma^* \to \Sigma^*, w \mapsto u$, where $w = uv$, $u = \varepsilon \vee u \in \Sigma^* \cdot I$ and $v \in O^*$
- $cont: \Sigma^* \to \mathfrak{P}(\Sigma^*), w \mapsto \{wu \mid u \in \Sigma^*\}$

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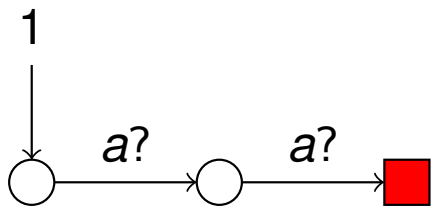
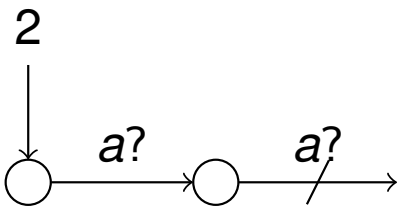
Missing Inputs





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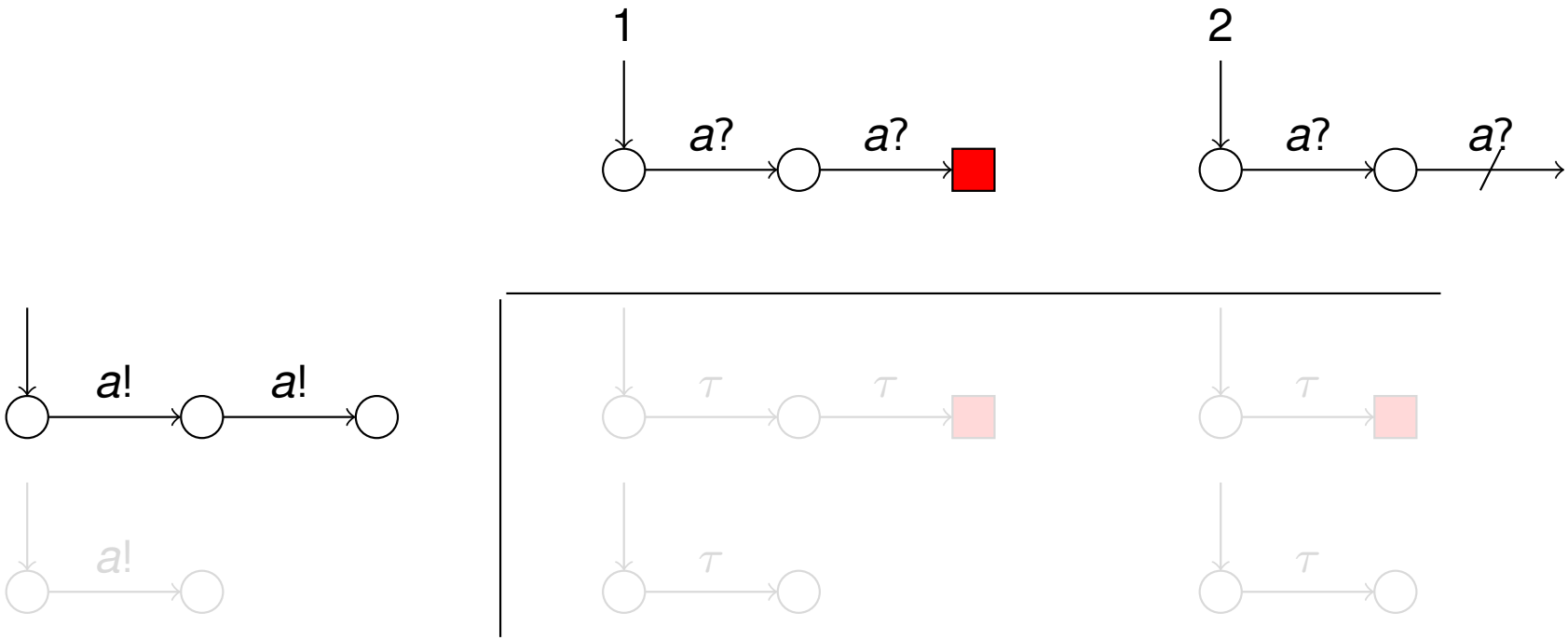

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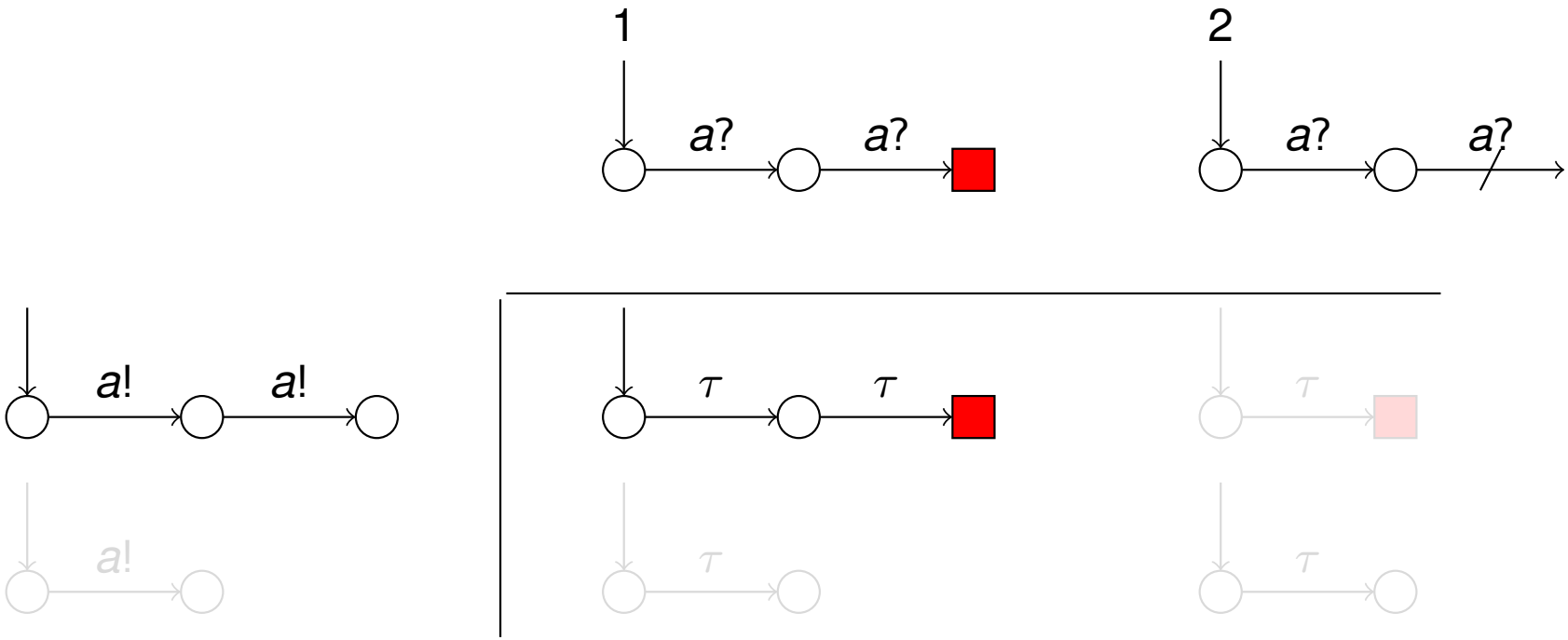

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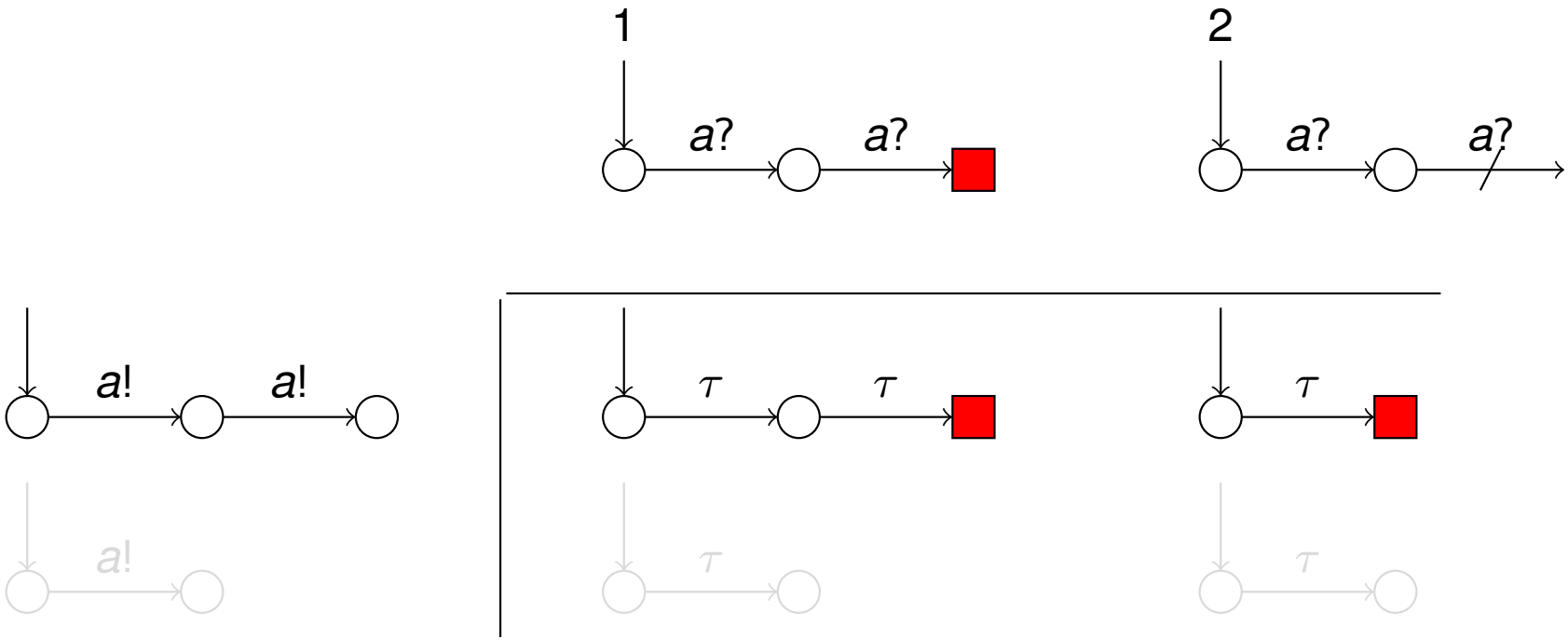

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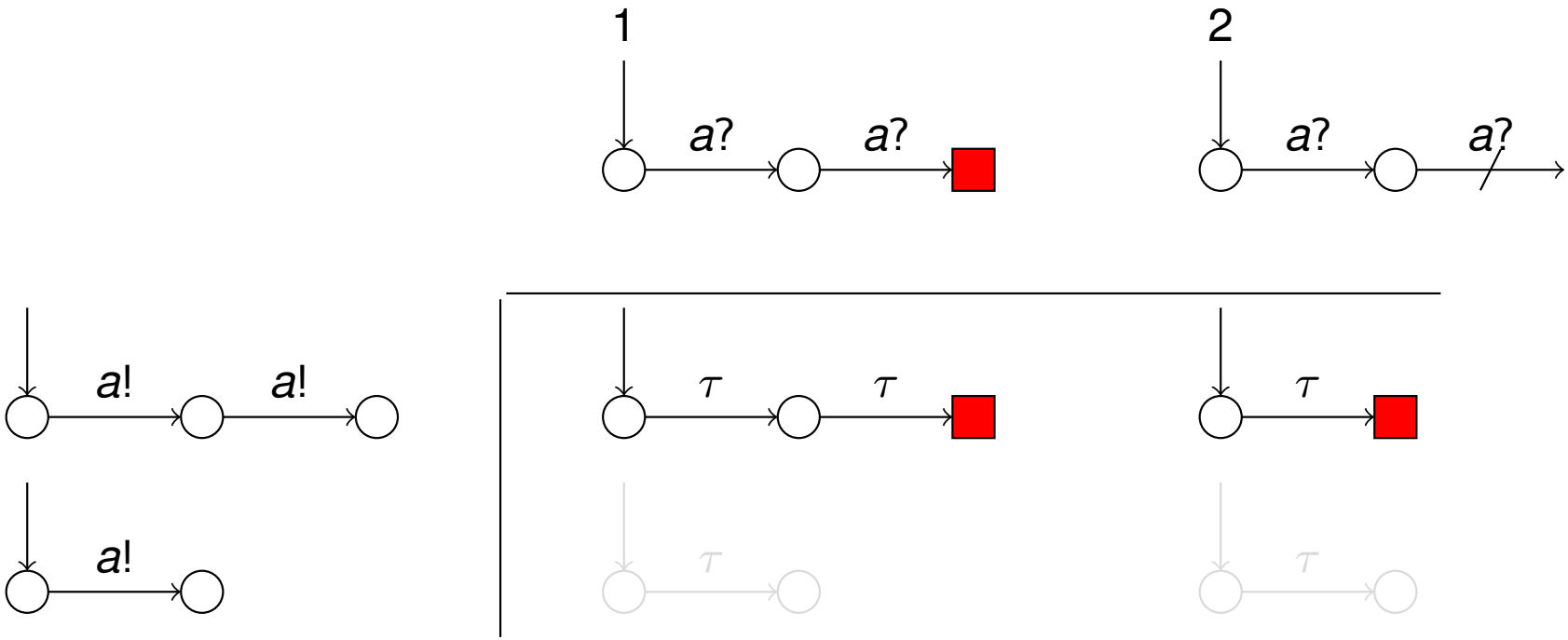

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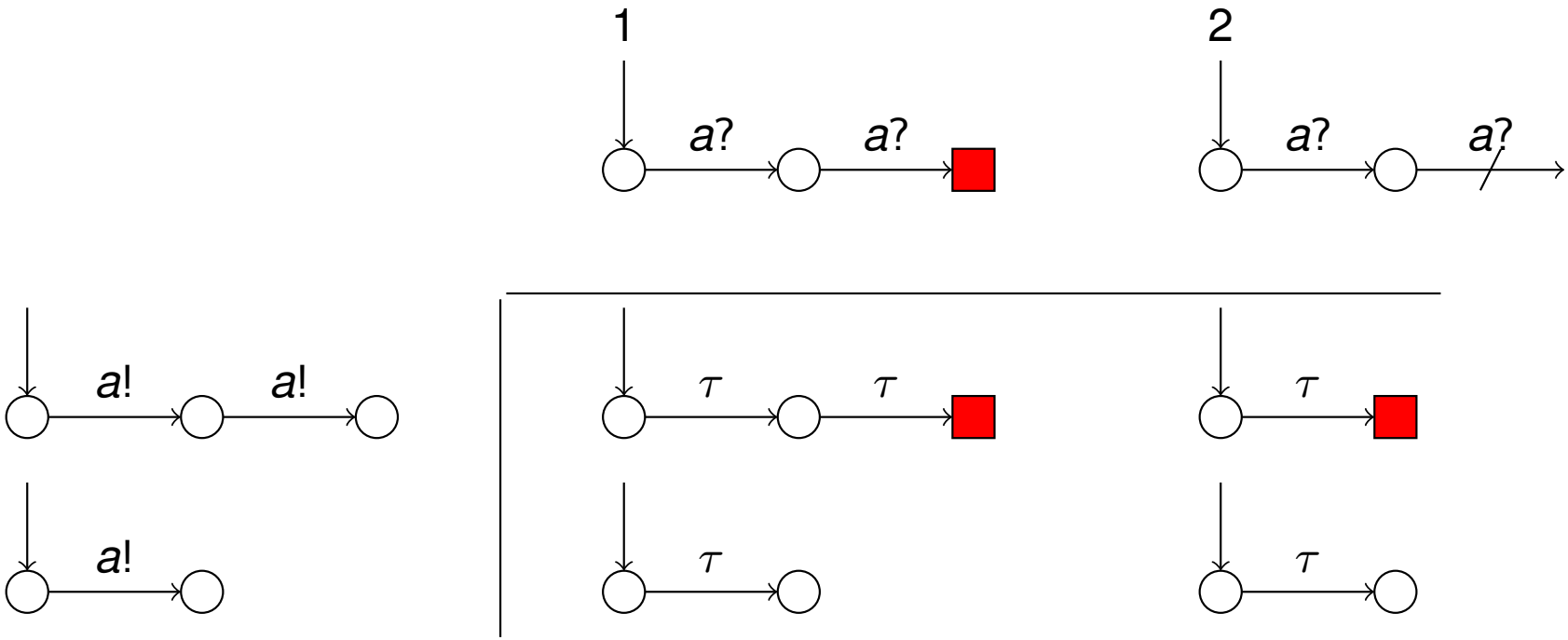

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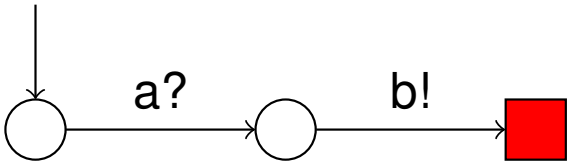

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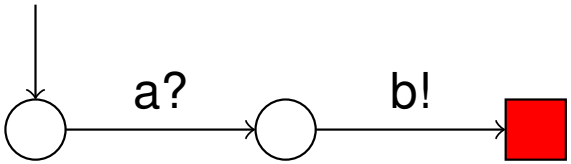

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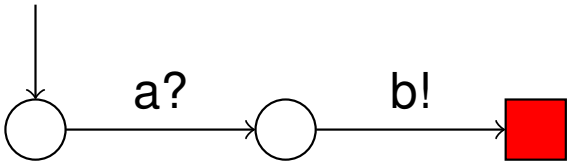

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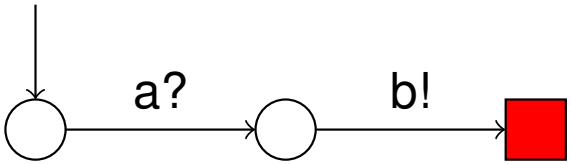

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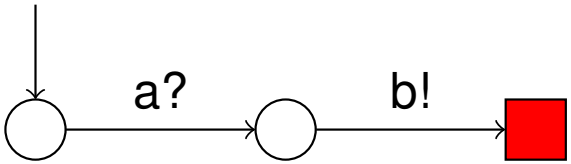

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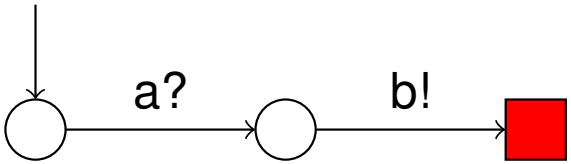

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Notion of error: $\varepsilon \in ET(S)$
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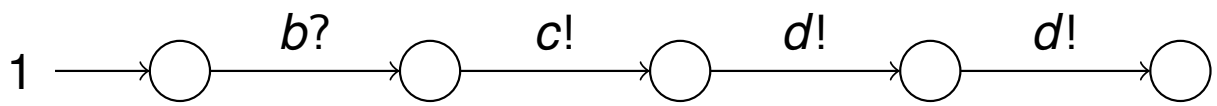

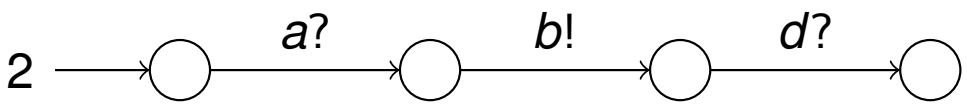

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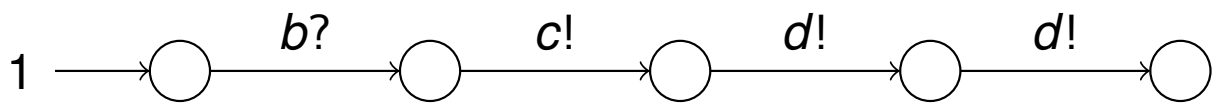

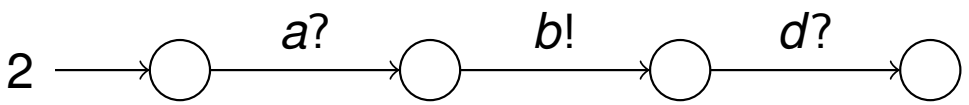

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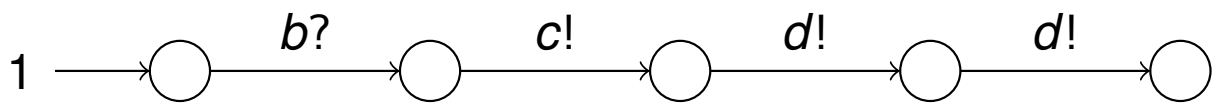

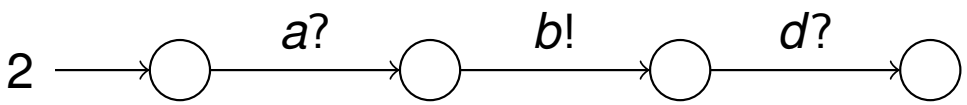

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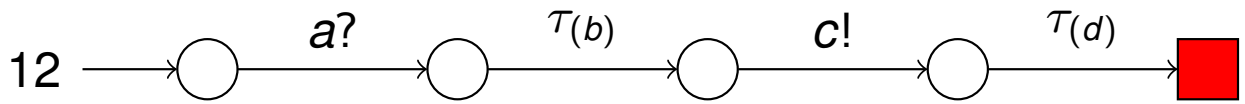

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Proove $Impl \sqsubseteq^{c}_{loc} Spec \Rightarrow Impl \sqsubseteq_{loc} Spec$

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- Associativity w.r.t. $=_{loc}$
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Notions of Error

Basic requirement:
If the specification is error free, then the implementation has to be error free as well

Notions of having an error:

1. An error state is reachable by locally controlled actions (internal and output)
2. An error state is reachable by internal actions
3. An error state is reachable at all

Internal errors

S has an error iff $q_0 \stackrel{\varepsilon}{\Rightarrow} q \in E$

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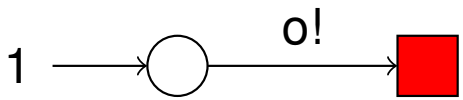

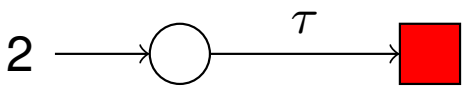

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- Pruned error pairs:
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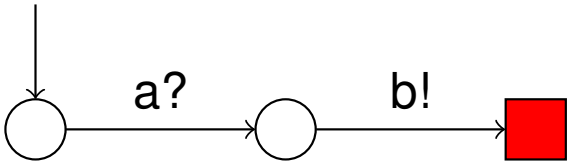

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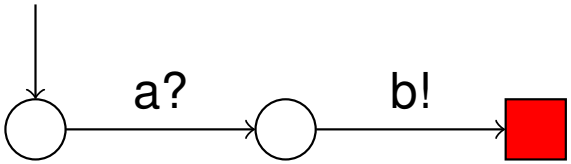

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Refinement preorder:

$$Impl \sqsubseteq_{int} Spec :\Leftrightarrow \begin{cases} EP(Impl) \subseteq EP(Spec) \\ EPL(Impl) \subseteq EPL(Spec) \end{cases}$$

Notion of error: $(\varepsilon, \emptyset) \in EP(S)$

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- $EP_{12} = cont\Big(prune\Big(\big(EP_1 \mid EPL_2\big) \cup \big(EPL_1 \mid EP_2\big)\Big)\Big)$
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- Both approaches use pruning
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Notions of Error

Basic requirement:
If the specification is error free, then the implementation has to be error free as well

Notions of having an error:

1. An error state is reachable by locally controlled actions (internal and output)
2. An error state is reachable by internal actions
3. An error state is reachable at all

Missing inputs

Again local semantics do not suffice: $ET_1 = \{aa\}\Sigma^* = ET_2$

Pessimistic Semantics

- Continued pruned traces:
$$CPT(S) = cont(prune(StT(S)));$$
- Flooded missing-input traces:
$$MIC(S) = MIT(S) \cup CPT(S);$$
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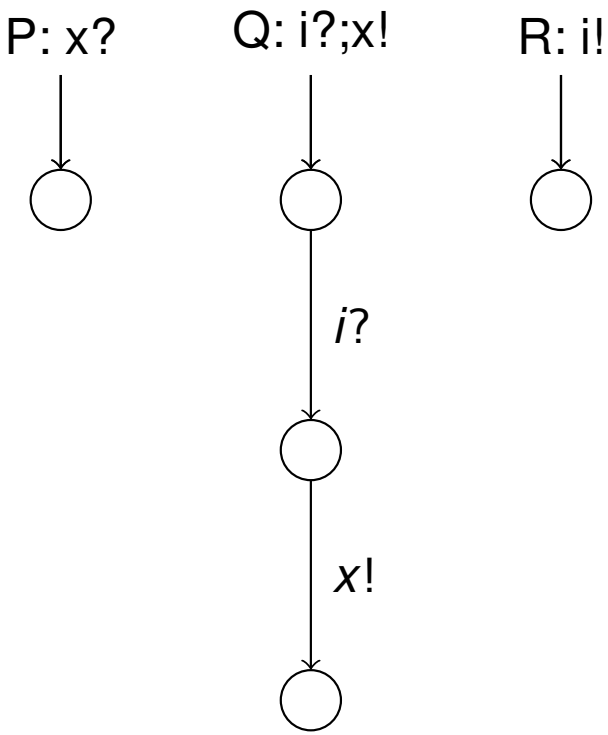

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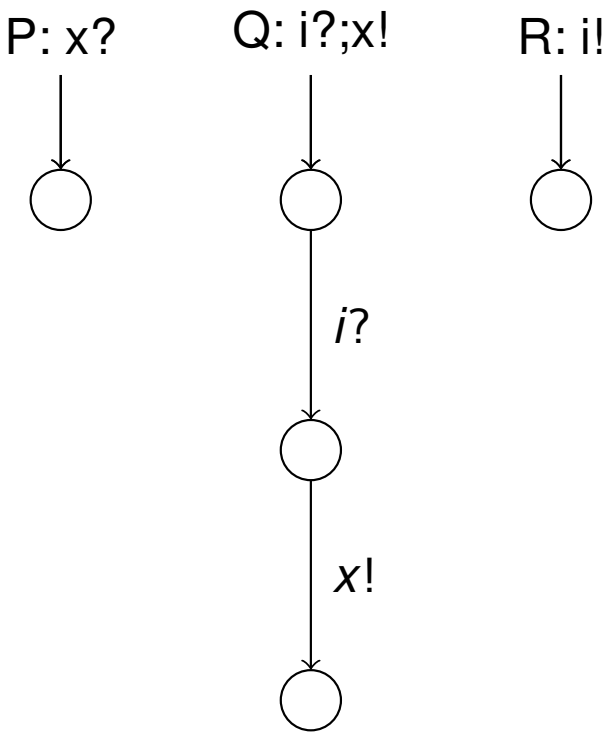

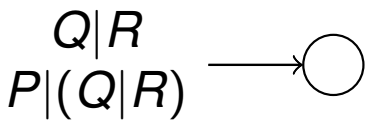

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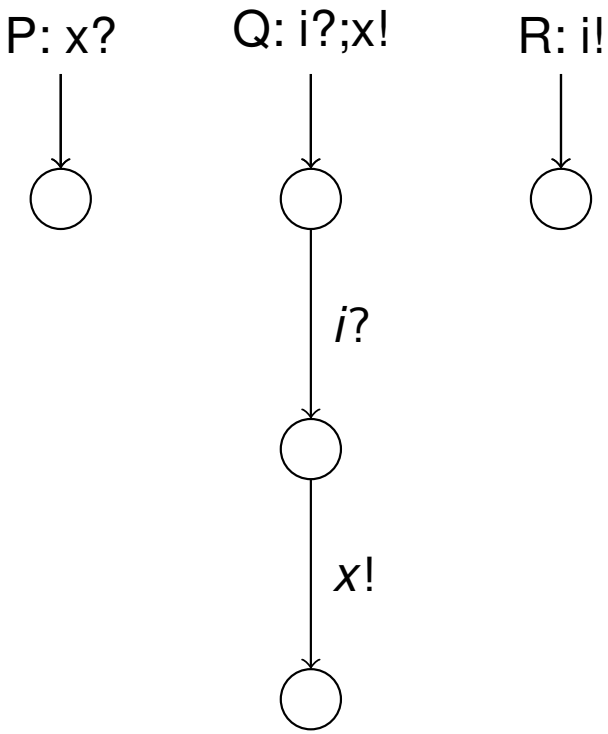
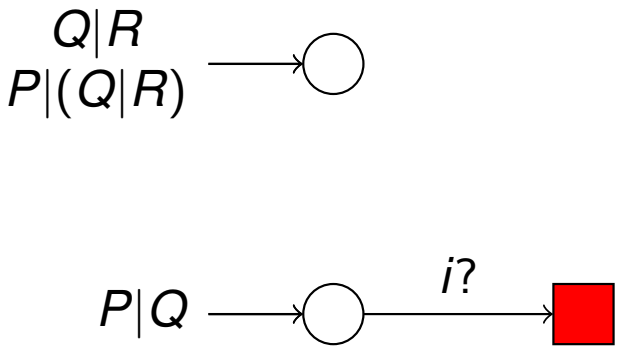
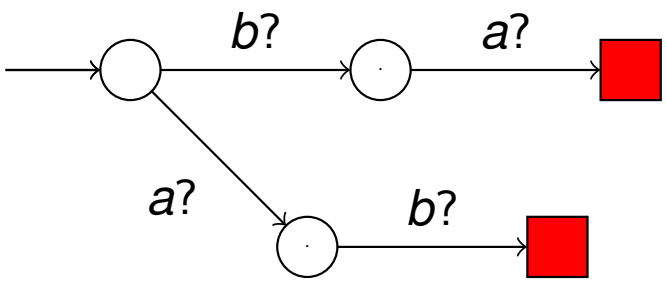
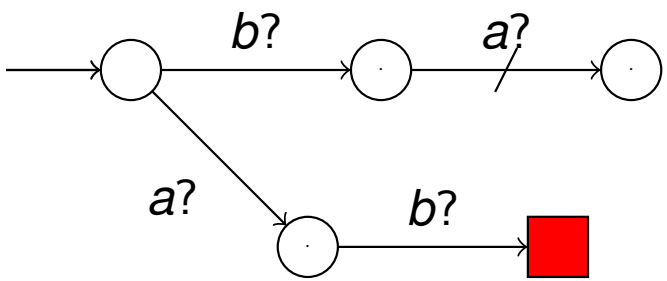




Non-Coarsestness





- $a, b \notin O_{Env}$: no change
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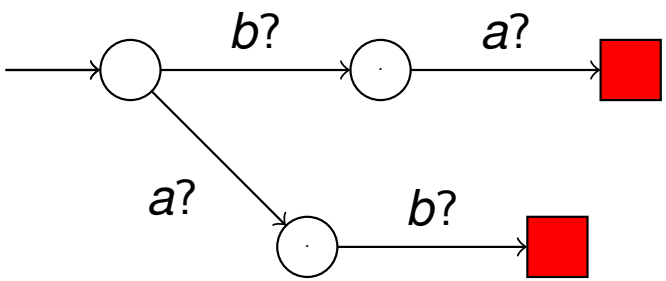

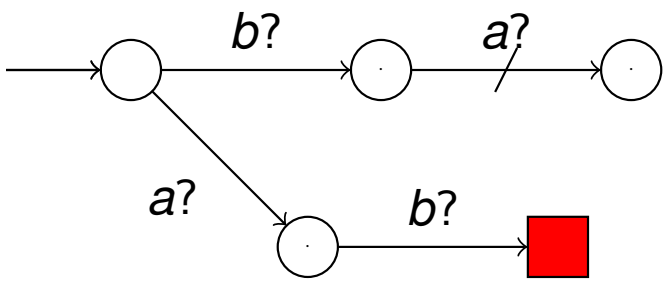

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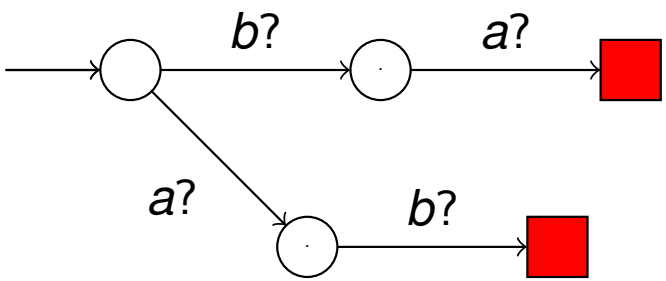

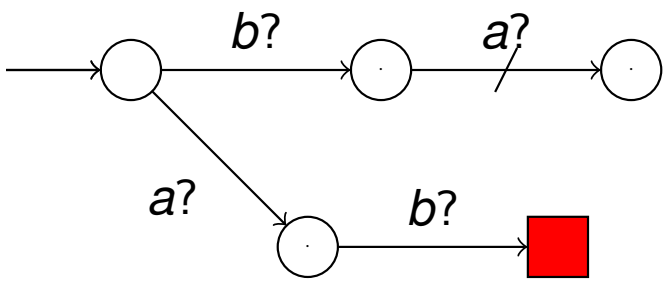

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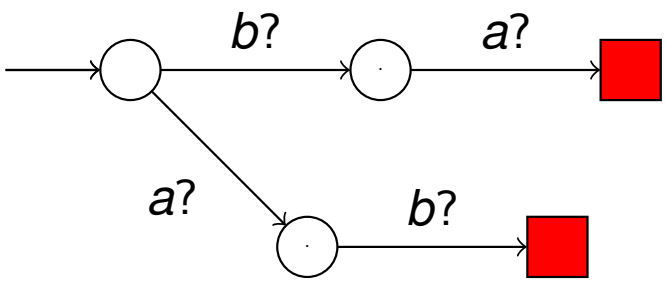

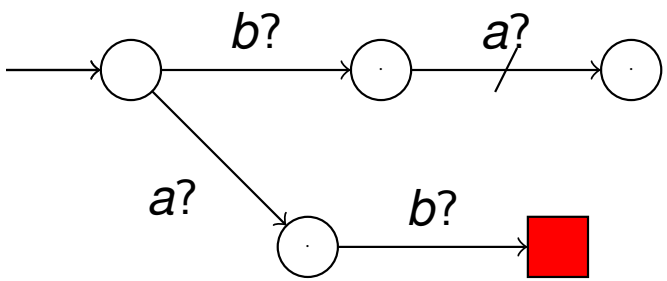

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Be optimistic!

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